

# Spinflation with backreaction

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**ABSTRACT:** We study brane inflation in flux compactifications at the nonlinear level, solving the D3-brane DBI-equations of motion including backreaction from the D3-D7 brane potential and from perturbations of the warp factor. We first numerically compute the exact functional form of the Kähler modulus valid on the entire supergravity background and obtain a two-field potential along radial and harmonic directions. We find that a valid perturbative expansion on the entire supergravity background with the Kähler modulus integrated out adiabatically in DBI inflation requires hierarchies of scales that determine compactification parameters different from those typical in slow-roll models. Our numerical results then show that the DBI inflationary solutions are quite robust against these nonlinear corrections.

**KEYWORDS:** String theory and cosmology; inflation.

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## 1. Introduction

The isotropy and homogeneity of the universe suggests that in very early times after the big bang the universe must have gone through a period of rapid exponential expansion called inflation. The quest for deriving inflation from a fundamental theory has led to the construction of inflationary models within the framework of string theory. For embedding inflation into string theory, one has to specify a string compactification whose low-energy effective theory contains a suitable inflaton field and potential. The many moduli fields that arise from Calabi-Yau compactifications can provide a large number of candidate scalar fields in the four-dimensional theory and any of these scalar fields may play the role of the inflaton field. However, moduli fields are generically either massless, or have a potential with runaway behaviour, which makes their interpretation as inflatons rather difficult. In addition to this, the computation of the effective potential in terms of all these scalar fields and degrees of freedom is a highly nontrivial task. Nevertheless, by considering the systematics of flux compactifications [1] and nonperturbative effects [2] in string theory, it is possible to reduce the degrees of freedom and stabilize all the moduli fields as required for a suitable model of inflation.

One particular way of constructing string inflationary models has been within the ‘KKLT’ framework, [2], according to which the older idea of brane inflation [3–6] is realised via the motion of a brane in the compact extra dimensions [7]. It was shown in [7] that the potential of the D3-brane in a Calabi-Yau flux compactification of type IIB theory [1] with the Kähler moduli stabilized according to [2] receives corrections that make the inflaton potential too steep for slow-roll inflation. It was

also conjectured in [7] that further compactification corrections to the inflaton potential may lead to a finely tuned cancellation of correction terms, so that the inflaton potential can be made flat. Major progress in this direction was made when these corrections were computed from ultraviolet deformations of the warped throat geometry and applied to explicit brane inflation models [8–22] (see also the reviews [23–29]). In these models the flattening of the inflaton potential is analysed under fine tuning and vast scanning of the compactification parameters. In particular, in the ‘delicate universe’ models studied in [12, 14, 15] it was shown that the inflaton potential can be made flat only in small region around an inflection point; elsewhere the potential remains steep. However, whether or not the potential can be made flat to realise slow-roll conditions, inflation can occur by the DBI effect with high speed and steep potentials [30, 31]. But the usual approach taken by the DBI brane inflation models in the literature is to consider brane motion in an imaginary self-dual (ISD) compactification in which the effects of moduli stabilization and backreaction are not included.

Any successful brane inflation model has to include the effects of compactification and moduli stabilization, which induce important corrections to the inflaton action. One particular correction from moduli stabilization is the departure of the ISD solution which induces harmonic dependence in the action of the probe D3-brane, [13–18], which has been studied to some extent in [19–21] and analysed in more detail in [32]. In previous work [32], we solved the DBI brane equations of motion in the warped deformed conifold [33] with harmonic dependent corrections from linearized perturbations around the ISD solution. We showed that just as angular motion increases the number of e-foldings (spinflation) [34–37], having additional angular dependence from linearized corrections also increases the number of e-foldings. However, this line of analysis considered multifield effects in D-brane inflation from only linearized perturbations around the ISD supergravity solution. In this paper, we extend our previous analysis [32] by including further harmonic dependent corrections from non-linear perturbations around the ISD solution, which also contribute to the inflaton action. In the noncompact limit, non-linear perturbations are dominated by imaginary anti-self-dual (IASD) fluxes sourced by moduli stabilizing wrapped D7-branes and the flux induced potential for the probe D3-brane in ten-dimensional supergravity equals the nonperturbatively-generated D3-D7 potential in four-dimensional supergravity [18].

One particular motivation of extending our previous analysis [32] by including such nonperturbative corrections is the possibility of increasing the amount of inflation and decreasing the level of non-Gaussianity due to backreaction effects [17]. The backreaction on the mobile D3-brane is sourced by itself [13]. The D3-brane in a flux compactification containing holomorphically embedded D7-branes corrects the warp factor which in turn corrects the warped volume of four-cycles wrapped by D7-branes. This then corrects the D3-D7 potential causing the backreaction on the D3-brane.

The perturbations of the warp factor are given by the Green’s function and correct the  $\gamma_{\text{DBI}}$ -factor which controls the level of non-Gaussianity. The Green’s function solves the noncompact supergravity equation of motion describing the perturbations around the ISD solution sourced by IASD fluxes and may be expanded in an infinite set of eigenstates of the Laplacian containing harmonic dependent hypergeometric functions [38, 39].

In this paper we solve the D3-brane equations of motion from the DBI action in the warped deformed conifold [33] with harmonic dependence from the leading correction terms of the warp factor and the D3-D7 potential. To solve the brane equations of motion, we note that the D3-brane potential including nonperturbative corrections depends on the functional form of the Kähler modulus and that of the D7-brane embedding. For the Kuperstein embedding of D7-branes [40] and a simple choice of an harmonic dependent trajectory on the deformed conifold, we integrate out the Kähler modulus and reduce the multifield D3-brane potential to a simple two-field potential depending on one radial and one harmonic direction of the conical geometry. In our numerical integrations we integrate out the Kähler modulus by computing its exact functional form valid on both the infrared (IR) and ultraviolet (UV) regions of the supergravity background. We find that computing the Kähler modulus within the adiabatic approximation in DBI inflation requires certain hierarchies of scales which determine the set of compactification parameters different from those in slow-roll models. Integrating the brane equations of motion for the consistent choice of parameters with the numerically computed Kähler modulus shows that the DBI inflationary solutions are quite robust against non-linear harmonic dependent corrections from perturbations of the warp factor and the D3-D7 brane potential.

The paper is organized as follows. In section 2, we outline our supergravity background and specialize to the warped deformed conifold. In section 3, we consider the general set up for brane inflation in the warped deformed conifold deriving the explicit form of our D3-brane potential and its relating DBI brane equations of motion. In section 4, we present our numerical results. We first determine the relevant parameter space and compute the Kähler modulus. We then solve the DBI brane equations of motion. In section 5, we conclude with a brief discussion.

## 2. Type IIB supergravity

The setting that we choose for our brane inflationary analysis is the Calabi-Yau flux compactification of type IIB theory containing a warped throat region generated by fluxes [1]. We therefore consider backgrounds in low-energy IIB supergravity, which in the Einstein frame can be represented by the action

$$S_{\text{IIB}} = \frac{1}{2\kappa_{10}^2} \left\{ \int d^{10}x \sqrt{|g|} \left[ \mathcal{R}_{10} - \frac{|\partial\tau|^2}{2(\text{Im}\tau)^2} - \frac{|G_3|^2}{12\text{Im}\tau} - \frac{|\tilde{F}_5|^2}{4 \cdot 5!} \right] + \frac{1}{4i} \int \frac{C_4 \wedge G_3 \wedge G_3^*}{\text{Im}(\tau)} + S_{\text{loc}} \right\}. \quad (2.1)$$

Here  $S_{\text{loc}}$  stands for localized contributions from D-brane and orientifold planes;  $\tau = C_0 + ie^{-i\phi}$  is the axion-dilaton field;  $G_3 = F_3 - \tau H_3$  is the combination of the R–R and NS–NS three-form fluxes  $F_3 = dC_2$  and  $H_3 = dB_2$  (with  $C_2$  and  $B_2$  being the R–R and NS–NN two-forms, respectively);  $C_4$  is the R–R four-form;  $\tilde{F}_5 = dC_4 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3$  is the self-dual five-form,  $\tilde{F}_5 = \star_{10}\tilde{F}_5$ , with  $\star_{10}$  denoting the ten-dimensional Hodge-star operator;  $\mathcal{R}_{10}$  is the ten-dimensional Ricci-scalar;  $\kappa_{10}^2 = \frac{1}{2}(2\pi)^7\alpha'^4 g_s^2$  is the ten-dimensional gravitational coupling.

In a flux compactification we may take the line element of the form

$$ds_{10}^2 = h^{-1/2}(y) ds_4^2 - h^{1/2}(y) ds_6^2, \quad (2.2)$$

where  $h$  is the warp factor depending only on the internal coordinates  $y := y^m$ ,  $ds_6^2$  is the metric on the internal manifold and  $ds_4^2$  is the four-dimensional flat metric, which will be the FRW metric for cosmological analysis.

Following [1], we take the self-dual five-form to be given by

$$\tilde{F}_5 = (1 + \star_{10}) \left[ d\alpha(y) \wedge dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \right], \quad (2.3)$$

in the Poincare invariant case, where  $\alpha(y)$  is a function of the internal coordinates. The Einstein equations and five-form Bianchi identity then imply

$$\Delta_{(0)}\Phi_{\pm} = \frac{e^{\Phi}}{24h^2} |G_{\pm}|^2 + h|\nabla\Phi_{\pm}| + \mathcal{R}_4 + \text{local} \quad (2.4)$$

where  $\Delta_{(0)}$  is the Laplacian with respect to the six-dimensional unperturbed Calabi-Yau metric  $g_{mn}^{(0)}$ , and

$$G_{\pm} \equiv (i \pm \star_6)G_3, \quad \Phi_{\pm} \equiv \frac{1}{h} \pm \alpha. \quad (2.5)$$

The equation of motion for the three-form flux is

$$d\Lambda + \frac{i}{2} \frac{d\tau}{\text{Im}(\tau)} \wedge (\Lambda + \bar{\Lambda}) = 0, \quad (2.6)$$

where by definition

$$\Lambda \equiv \Phi_+ G_- + \Phi_- G_+. \quad (2.7)$$

For  $G_- = 0$ , i.e.  $\star_6 G_3 = iG_3$ , the flux  $G_3$  is imaginary self-dual (ISD), the background metric is Calabi-Yau with the five-form flux given by  $\alpha = h^{-1}$  [1]. The background that satisfies these conditions is called ISD. The specific example of such a background that we are interested in is the warped deformed conifold [33]. The deformed conifold is a noncompact and nonsingular Calabi-Yau three-fold in  $\mathbb{C}^4$  defined by the following constraint equation:

$$\sum_{A=1}^4 (z_A)^2 = \epsilon^2. \quad (2.8)$$

where  $\{z_A, A = 1, 2, 3, 4\}$  represent the local complex coordinates in  $\mathbb{C}^4$ , and  $\epsilon$  is the deformation parameter which can be made real by phase rotation. For vanishing  $\epsilon$ , Eq. (2.8) gives the singular conifold and describes a cone over a five-dimensional Einstein manifold  $X_5$ . For us the nonsingular limit is relevant in which  $X_5$  is the  $[SU(2) \times SU(2)]/U(1)$  coset space  $T^{1,1}$  of topology  $S^2 \times S^3$  parametrised by a set of five Euler angles  $\Psi = \{\theta_i, \varphi_i, \psi\}$  with  $0 \leq \theta_i \leq \pi$ ,  $0 \leq \varphi_i \leq 2\pi$ ,  $0 \leq \psi \leq 4\pi$  ( $i = 1, 2$ ), and the would-be singularity at the tip,  $r = 0$ , is replaced by a blown-up  $S^3$  of  $T^{1,1}$  amounting to the deformation measured by  $\epsilon$ . The base of the cone can be parametrised by the coordinates  $y_i$  in a standard way [41]

$$y_1 = \frac{1}{\sqrt{2}} \left( \cos \frac{\theta_1}{2} \cos \frac{\theta_2}{2} e^{\frac{i}{2}(\varphi_1 + \varphi_2 + \psi)} - \sin \frac{\theta_1}{2} \sin \frac{\theta_2}{2} e^{-\frac{i}{2}(\varphi_1 + \varphi_2 - \psi)} \right), \quad (2.9)$$

$$y_2 = \frac{i}{\sqrt{2}} \left( \cos \frac{\theta_1}{2} \cos \frac{\theta_2}{2} e^{\frac{i}{2}(\varphi_1 + \varphi_2 + \psi)} + \sin \frac{\theta_1}{2} \sin \frac{\theta_2}{2} e^{-\frac{i}{2}(\varphi_1 + \varphi_2 - \psi)} \right), \quad (2.10)$$

$$y_3 = -\frac{1}{\sqrt{2}} \left( \cos \frac{\theta_1}{2} \sin \frac{\theta_2}{2} e^{\frac{i}{2}(\varphi_1 - \varphi_2 + \psi)} + \sin \frac{\theta_1}{2} \cos \frac{\theta_2}{2} e^{\frac{i}{2}(\varphi_2 - \varphi_1 + \psi)} \right), \quad (2.11)$$

$$y_4 = \frac{i}{\sqrt{2}} \left( \cos \frac{\theta_1}{2} \sin \frac{\theta_2}{2} e^{\frac{i}{2}(\varphi_1 - \varphi_2 + \psi)} - \sin \frac{\theta_1}{2} \cos \frac{\theta_2}{2} e^{\frac{i}{2}(\varphi_2 - \varphi_1 + \psi)} \right). \quad (2.12)$$

In terms of these, the coordinates of the deformed conifold in (2.8) can be written as

$$\begin{aligned} z_1 &= \frac{\epsilon}{\sqrt{2}} (e^{\frac{\eta}{2}} y_1 + e^{-\frac{\eta}{2}} \bar{y}_1), & z_2 &= \frac{\epsilon}{\sqrt{2}} (e^{\frac{\eta}{2}} y_2 + e^{-\frac{\eta}{2}} \bar{y}_2), \\ z_3 &= \frac{\epsilon}{\sqrt{2}} (e^{\frac{\eta}{2}} y_3 + e^{-\frac{\eta}{2}} \bar{y}_3), & z_4 &= \frac{\epsilon}{\sqrt{2}} (e^{\frac{\eta}{2}} y_4 + e^{-\frac{\eta}{2}} \bar{y}_4), \end{aligned} \quad (2.13)$$

where  $\eta$  is the ‘radial coordinate’. The six-dimensional Calabi-Yau metric,  $ds_6^2$ , on the deformed conifold can be obtained from the Kähler potential given as [41]

$$k(\eta) = \frac{\epsilon^{4/3}}{2^{1/3}} \int_0^\eta d\eta' [\sinh(2\eta') - 2\eta']^{1/3}. \quad (2.14)$$

Here we note that this Kähler potential is derived from the relation

$$r^3 = \sum_{A=1}^4 |z_A|^2 = \epsilon^2 \cosh \eta, \quad (2.15)$$

and the metric for the deformed conifold reads as [41]

$$\begin{aligned} ds_6^2 = & \frac{1}{2} \epsilon^{4/3} K(\eta) \left[ \frac{1}{3K(\eta)^3} \{d\eta^2 + (\omega^5)^2\} + \cosh^2 \frac{\eta}{2} \left\{ \frac{1}{2} \sum_{i=1}^4 (\omega^i)^2 + \sum_{i \neq j=1}^4 \omega^i \omega^j \right\} \right. \\ & \left. + \sinh^2 \frac{\eta}{2} \left\{ \frac{1}{2} \sum_{i=1}^4 (\omega^i)^2 - \sum_{i \neq j=1}^4 \omega^i \omega^j \right\} \right], \end{aligned} \quad (2.16)$$

where

$$\begin{aligned} \omega^1 &= -\sin \theta_1 d\varphi_1, & \omega^2 &= d\theta_1, & \omega^3 &= \cos \psi \sin \theta_2 d\varphi_2 - \sin \psi d\theta_2, \\ \omega^4 &= \sin \psi \sin \theta_2 d\varphi_2 + \cos \psi d\theta_2, & \omega^5 &= d\psi + \cos \theta_1 d\varphi_1 + \cos \theta_2 d\varphi_2, \end{aligned} \quad (2.17)$$

and

$$K(\eta) = \frac{(\sinh(2\eta) - 2\eta)^{1/3}}{2^{1/3} \sinh \eta}. \quad (2.18)$$

The proper radial coordinate which measures the actual distance up the throat in the six-dimensional metric is given by

$$r(\eta) = \frac{\epsilon^{2/3}}{\sqrt{6}} \int_0^\eta \frac{dx}{K(x)}. \quad (2.19)$$

The warping in this background is induced by the presence of type IIB background fluxes including  $M$  units of  $F_3$  flux through the cycle  $A$  and  $-K$  units of  $H_3$  flux through the cycle  $B$ . Due to the presence of three-form fluxes the above background emerges as a solution to Einstein's equations and the resulting supergravity background is described by a throat with its tip being located at a finite radial coordinate  $r_{\text{IR}}$  while at  $r_{\text{UV}}$  the throat is glued into an unwarped bulk geometry. The warp factor is given in terms of the  $\eta$  coordinate as [33]

$$h_{\text{KS}} = 2(g_s M \alpha')^2 \epsilon^{-8/3} I(\eta), \quad (2.20)$$

$$I(\eta) \equiv \int_\eta^\infty dx \frac{x \cosh x - 1}{\sinh^2 x} (\sinh x \cosh x - x)^{1/3}. \quad (2.21)$$

At small and large radius, the warp factor takes the form:

$$h_{\text{KS}} = \begin{cases} 2(g_s M \alpha')^2 \epsilon^{-8/3} \left( 0.5699 - 2\epsilon^{-4/3} \frac{r^2}{3} \right); & \eta \rightarrow \text{small} \\ \frac{27}{8} \frac{(g_s M \alpha')^2}{r^4} \left( \ln \frac{r^3}{\epsilon^2} + \ln \frac{4\sqrt{2}}{3\sqrt{3}} - \frac{1}{4} \right); & \eta \rightarrow \text{large} \end{cases}$$

In an ISD compactification such as the warped deformed conifold, only the complex-structure moduli are stabilized but not the Kähler moduli. Embedding the warped throat into the Calabi-Yau sector with Kähler moduli stabilized breaks the no-scale structure and induces perturbations around the ISD solution. In the noncompact limit ( $M_{\text{pl}} \rightarrow \infty$ ), these perturbations are sourced by non-linear effects including only IASD fluxes. These fluxes satisfy the IASD conditions [17]

$$d\Lambda = 0, \quad \text{where} \quad \star_6 \Lambda = -i\Lambda \quad (2.22)$$

on general Calabi-Yau cones. The perturbations of  $\Phi_-$  around ISD conditions containing IASD fluxes as the dominant source satisfy the second-order supergravity equation of motion [17, 18]

$$\Delta_{(0)} \Phi_- = \frac{g_s}{96} |\Lambda|^2, \quad (2.23)$$

where  $\Delta_{(0)}$  is the Laplacian obtained from the unperturbed Calabi-Yau metric  $g_{mn}^{(0)}$ , as before. The solution of Eq. (2.23) can always be written as the sum of first order homogeneous and second order inhomogeneous solution in the form [17, 18]:

$$\Phi_-(y) = \Phi_-^{(1)}(y) + \Phi_-^{(2)}(y), \quad (2.24)$$

where the homogeneous solution satisfies the Laplace equation

$$\Delta_{(0)} \Phi_-^{(1)}(y) = 0. \quad (2.25)$$

A particularly simple solution of the Laplace equation on the deformed conifold takes the form [32]

$$\Phi_-^{(1)}(\eta, \theta) \propto (\cosh \eta \sinh \eta - \eta)^{1/3} \cos \theta. \quad (2.26)$$

The inhomogeneous solution can always be written in the form:

$$\Phi_-^{(2)}(y) = \frac{g_s}{96} \int d^6 y' \mathcal{G}(y; y') |\Lambda|^2(y'), \quad (2.27)$$

where

$$\Delta_{(0)} \mathcal{G}(y; y') = \delta(y - y'). \quad (2.28)$$

Here and in what follows  $y$  denotes a collective internal coordinate consisting of the radial and six angular directions, where the later will always be denoted by  $\Psi$ . On a



general Calabi-Yau cone with Kähler form  $J$  and holomorphic (3,0) form  $\Omega$ , we may turn on (1,2) flux,  $\Lambda_1$ , and a non-primitive (2,1) flux,  $\Lambda_2$  [17, 18]:

$$\Lambda_1 = \partial\bar{\partial}f_1\bar{\Omega}, \quad \Lambda_2 = \partial f_2 \wedge J, \quad (2.29)$$

with  $f_1$  and  $f_2$  being holomorphic functions. The solution (2.27) takes the form [17, 18]:

$$\Phi_-^{(2)}(y) = \frac{g_s}{32} \left[ \mathcal{K}^{\Sigma\bar{\Sigma}} \partial_\Sigma f_1 \bar{\partial}_{\bar{\Sigma}} f_1 + 2|f_2|^2 \right], \quad (2.30)$$

where  $\mathcal{K}^{\Sigma\bar{\Sigma}}$  is the Kähler metric. For a specific choice of the  $f_i$ 's, Eq. (2.30) determines the explicit solution of Eq. (2.23) up to harmonic terms.

In the IR region of the throat where  $\eta$  is small, the Green's function is that of the deformed conifold and takes the form [39]

$$\mathcal{G}(y; y') = \mathcal{G}(\eta, y_4; \mathbf{e}_0) = -\frac{3^{2/3}}{2^{8/3}\pi^3\epsilon^{8/3}} \sum_{j=0, \frac{1}{2}, 1, \dots} \sum_{m=-j}^j \frac{\sqrt{2j+1}}{\eta} \mathcal{F}_{jm}(y_4, \bar{y}_4), \quad (2.31)$$

where  $\mathbf{e}_0$  parametrizes the blown up  $S^3$  at  $\eta = 0$  and  $\mathcal{F}_{jm}$  stand for the hypergeometric functions given by

$$\begin{aligned} \mathcal{F}_{0,0} &= 1, & \mathcal{F}_{\frac{1}{2}, \frac{1}{2}} &= 2y_4, & \mathcal{F}_{\frac{1}{2}, -\frac{1}{2}} &= 2\bar{y}_4, & \mathcal{F}_{1,1} &= 2\sqrt{3}y_4^2, \\ \mathcal{F}_{1,0} &= -\sqrt{3}(1 - 4y_4\bar{y}_4), & \mathcal{F}_{1,-1} &= 2\sqrt{3}\bar{y}_4^2, & \mathcal{F}_{\frac{3}{2}, \frac{3}{2}} &= 4\sqrt{2}y_4^3, \\ \mathcal{F}_{\frac{3}{2}, \frac{1}{2}} &= -4\sqrt{2}y_4(1 - 3y_4\bar{y}_4), & \mathcal{F}_{\frac{3}{2}, -\frac{1}{2}} &= -4\sqrt{2}\bar{y}_4(1 - 3y_4\bar{y}_4), \\ \mathcal{F}_{\frac{3}{2}, \frac{3}{2}} &= 4\sqrt{2}\bar{y}_4^3. \end{aligned} \quad (2.32)$$

In the UV region of the throat where  $\eta$  is large, we may introduce another radial coordinate  $r$  through  $r^3 \sim \epsilon^2 e^\eta$  (see Eq. (2.15)) and the Green's function is that of the singular conifold given by [13]

$$\mathcal{G}(y; y') = \sum_L \frac{Y_L^*(\Psi') Y_L(\Psi)}{2\sqrt{\Lambda_L + 4}} \times \begin{cases} \left( \frac{1}{r'^4} \left( \frac{r}{r'} \right) \right)^{c_L^+} & r \leq r' \\ \left( \frac{1}{r^4} \left( \frac{r'}{r} \right) \right)^{c_L^+} & r \geq r' \end{cases} \quad (2.33)$$

with the harmonic eigenfunctions

$$Y_L(\Psi) = Z_{j_1, m_1, R}(\theta_1) Z_{j_2, m_2, R}(\theta_2) e^{im_1\varphi_1 + im_2\varphi_2} e^{\frac{i}{2}R\psi}, \quad (2.34)$$

$$Z_{j_i, m_i, R}^{\text{I}}(\theta_i) = N_L^{\text{I}} (\sin \theta_i)^{m_i} \left( \cot \frac{\theta_i}{2} \right)^{R/2} \times {}_2F_1 \left( -j_i + m_i, 1 + j_i + m_i, 1 + m_i - \frac{R}{2}; \sin^2 \frac{\theta_i}{2} \right), \quad (2.35)$$

$$Z_{j_i, m_i, R}^{\text{II}}(\theta_i) = N_L^{\text{II}} (\sin \theta_i)^{R/2} \left( \cot \frac{\theta_i}{2} \right)^{m_i} \times {}_2F_1 \left( -j_i + \frac{R}{2}, 1 + j_i + \frac{R}{2}, 1 - m_i + \frac{R}{2}; \sin^2 \frac{\theta_i}{2} \right). \quad (2.36)$$

The normalization factors  $N_L^{\text{I/II}}$  relation is given by

$$V_{T^{1,1}} \int_0^1 dx [Z_{j_1, m_1, R}(x)]^2 \int_0^1 dy [Z_{j_2, m_2, R}(y)]^2 = 1. \quad (2.37)$$

In the above relations  ${}_2F_1(a, b, c; d)$  stands for hypergeometric functions;  $L$  is a multi-index with the data  $L \equiv (j_1, j_2), (m_1, m_2), R$ , where  $j_1$  and  $j_2$  are both integers or half-integers with  $m_1 \in \{-j_1, \dots, j_1\}$  and  $m_2 \in \{-j_2, \dots, j_2\}$ ;  $\Lambda_L = 6(j_1(j_1 + 1) + j_2(j_2 + 1) - R^2/8)$  denotes the spectrum of the full wave function and the eigenfunctions transform under  $SU(2)_1 \times SU(2)_2$  as the spin  $(j_1, j_2)$  representation and under the  $U(1)_R$  with charge  $R$ ;  $c_L^\pm \equiv -2 \pm \sqrt{\Lambda_L + 4}$ .

### 3. Multifield D-brane inflation

We now embed a mobile D3-brane in the supergravity background described in the previous section and analyse its the four-dimensional effective action. In the supergravity background with metric ansatz (2.2) the effective action takes the form

$$I = \frac{M_{\text{Pl}}}{2} \int d^4x \sqrt{-g} \mathcal{R} - g_s^{-1} \int d^4x \sqrt{-g} \left[ h^{-1} (\gamma_{\text{DBI}}^{-1} - 1) + V(\phi^m) \right], \quad (3.1)$$

$$\gamma_{\text{DBI}}^{-1} = \sqrt{1 - h g_{mn} g^{\mu\nu} \partial_\mu \phi^m \partial_\nu \phi^n}.$$

Here  $g_s$  is the string coupling, and  $M_{\text{Pl}}$  is the Planck-mass. The first term in this action is the ordinary four-dimensional Einstein-Hilbert action, which arises from dimensional reduction of the closed string sector of the ten-dimensional action. The second part contains the action that controls the dynamics of the fields, parametrizing the position of the brane along the internal coordinates,  $\phi^m$ . In a strongly warped region,  $h \gg 1$ , the kinetic energy's pre-factor of  $h^{-1}$  in Eq. (3.1) suppresses it relative to  $V(\phi^m)$  even when the motion is relativistic.

To study brane inflation, we will take the position of the D3-brane to be homogeneous,  $\phi^m = \phi^m(t)$ , and we will consider the four-dimensional metric to be the

standard unperturbed FRW metric:

$$ds_4^2 = g_{\mu\nu} dx^\mu dx^\nu = dt^2 - a^2(t) d\mathbf{x}^2, \quad (3.2)$$

where  $a(t)$  is the scale factor. The variation of the later part of the action (3.1) with respect to the metric produces the energy-momentum tensor, which has the form of a perfect fluid with its energy density and pressure given by:

$$E = T_3(h^{-1}[\gamma_{\text{DBI}} - 1] + V), \quad (3.3)$$

$$P = T_3(h^{-1}[1 - \gamma_{\text{DBI}}^{-1}] + V). \quad (3.4)$$

Here  $T_3 = ((2\pi)^3 \alpha'^2)^{-1}$  is the D3-brane tension. The full equations of motion are:

$$H^2 = \frac{E}{3M_{\text{pl}}^2}, \quad (3.5)$$

$$\dot{H} = -\frac{(E + P)}{2M_{\text{pl}}^2}, \quad (3.6)$$

$$\frac{1}{a^3} \frac{d}{dt} \left[ a^3 \gamma_{\text{DBI}} g_{mn}^{(0)} \dot{\phi}^n \right] = -T_3 \gamma_{\text{DBI}} (\gamma_{\text{DBI}}^{-1} - 1)^2 \frac{\partial_m h}{2h^2} + \frac{\gamma_{\text{DBI}}}{2} \frac{\partial g_{ln}^{(0)}}{\partial \phi^m} \dot{\phi}^l \dot{\phi}^n - \partial_m V, \quad (3.7)$$

where  $g_{mn}^{(0)}$  is the unperturbed Calabi-Yau metric, as before. The Planck-mass,  $M_{\text{pl}}$ , appearing above is bounded by the UV-scale as [32]:

$$M_{\text{pl}}^2 > \frac{\epsilon^{4/3} g_s M^2 T_3}{6\pi} J(\eta_{\text{UV}}), \quad (3.8)$$

where  $J(\eta) = \int I(\eta) \sinh^2 \eta$  with  $I(\eta)$  given by (2.21).

The D3-brane potential appearing in the above equations of motion is:

$$T_3 V = \frac{1}{2} m^2 \phi^2 + T_3 \Phi_-. \quad (3.9)$$

Here the first term is a mass term generated for the canonical inflaton  $\phi = \sqrt{T_3} r(\eta)$ , which by (3.10) takes the form:

$$\phi = \epsilon^{2/3} \sqrt{\frac{T_3}{6}} \int_0^\eta \frac{dx}{K(x)}. \quad (3.10)$$

We note that the normalization is strictly dependent on the position of the brane, which affects the volume modulus (see e.g. [13–18]), and also on the trajectory of the brane, even in the slow roll approximation, due to the inflaton,  $\phi$ , being a sigma model field, [43–45]. The last term in (3.9) arises from perturbations around the noncompact (ISD) supergravity solution described by Eq. (2.23). The homogeneous

linearized solution of Eq. (2.23) is given by (2.26). The inhomogeneous non-linear solution of Eq. (2.23) given in its implicit form as (2.30) equals the nonperturbatively generated D3-brane F-term potential in four-dimensional supergravity [18]. The F-term potential takes the following form:

$$V_F = e^{\kappa^2 \mathcal{K}} \left[ D_\Sigma W \mathcal{K}^{\Sigma\bar{\Xi}} \overline{D_{\bar{\Xi}} W} - 3\kappa^2 W \overline{W} \right], \quad (3.11)$$

where  $\{z^\Sigma\} \equiv \{\rho, z_\alpha; \alpha = 1, 2, 3\}$  and  $D_\Sigma W = \partial_\Sigma W + \kappa^2 (\partial_\Sigma \mathcal{K}) W$  with  $\mathcal{K}$  and  $W$  denoting the Kähler potential and superpotential, and  $\kappa^2 = M_{\text{pl}}^{-2}$  with  $M_{\text{pl}}^2$  bounded by (3.8). From (3.8) one can see that large  $\eta_{\text{UV}}$  implies large  $M_{\text{pl}}$ . In the noncompact limit ( $M_{\text{pl}} \rightarrow \infty$ ), the F-term potential (3.11) takes the form

$$V_F = \mathcal{K}^{\Sigma\bar{\Xi}} \partial_\Sigma W \overline{\partial_{\bar{\Xi}} W}. \quad (3.12)$$

For any holomorphic function  $W$  on the conical geometry, we may turn on the flux

$$\Lambda_{\Sigma\bar{\Xi}\bar{\Gamma}} = \partial_\Sigma \partial_{\bar{\Gamma}} W \mathcal{K}^{\Upsilon\bar{\Theta}} \bar{\Omega}_{\bar{\Theta}\bar{\Xi}\bar{\Gamma}} = \partial \partial W \cdot \bar{\Omega}, \quad (3.13)$$

where  $\Omega$  is the holomorphic (3, 0)-form. Solving Eq. (2.23) for Eq. (3.13) gives [18]

$$T_3 \Phi_- = \mathcal{K}^{\Sigma\bar{\Xi}} \partial_\Sigma W \overline{\partial_{\bar{\Xi}} W}. \quad (3.14)$$

The Kähler potential,  $\mathcal{K}$ , depends on the complex Kähler modulus  $\rho = \sigma + i\chi$ , and on the D3-brane position,  $\{z_\alpha, \bar{z}_\alpha\}$ , [46]

$$\mathcal{K}(z^\alpha, \bar{z}^\alpha, \rho, \bar{\rho}) = -3\kappa^{-2} \log[\rho + \bar{\rho} - \gamma k(z^\alpha, \bar{z}^\alpha)] \equiv -3\kappa^{-2} \log U(z, \rho). \quad (3.15)$$

Here  $k(z^\alpha, \bar{z}^\alpha)$  is the so-called ‘little’ Kähler potential of the Calabi-Yau manifold,  $\kappa^2 = M_{\text{pl}}^{-2}$  as before, and  $\gamma$  is a normalization factor given by

$$\gamma = \frac{\sigma_0 T_3}{3M_{\text{pl}}^2} \quad (3.16)$$

where  $\sigma_0$  denotes the value of  $\sigma$  when the D3-brane is at its stabilized configuration [8, 14]. The Kähler metric and its inverse take the form [8]

$$\mathcal{K}_{\Xi\bar{\Sigma}} = \frac{3}{\kappa^2 U^2} \left( \begin{array}{c|c} 1 & -\gamma k_{\bar{\beta}} \\ \hline -\gamma k_\alpha & U \gamma k_{\alpha\bar{\beta}} + \gamma^2 k_\alpha k_{\bar{\beta}} \end{array} \right), \quad (3.17)$$

$$\mathcal{K}^{\Delta\bar{\Gamma}} = \frac{\kappa^2 U}{3} \left( \begin{array}{c|c} U + \gamma k_\gamma k^{\gamma\bar{\delta}} k_{\bar{\gamma}} & k_\gamma k^{\gamma\bar{\beta}} \\ \hline k^{\alpha\bar{\delta}} k_{\bar{\delta}} & \frac{1}{\gamma} k^{\alpha\bar{\beta}} \end{array} \right), \quad (3.18)$$

where  $k_{\alpha\bar{\beta}} \equiv \partial_\alpha \partial_{\bar{\beta}} k$  is the Calabi-Yau metric, and  $k_\alpha \equiv k_{,\alpha}$ . The superpotential  $W$ , also depends on the D3-brane positions,  $\{z_\alpha\}$ , and is given by

$$W(\rho) = W_0 + A(z^\alpha)e^{-a\rho}. \quad (3.19)$$

The first part of the superpotential is the Gukov-Vafa perturbative superpotential [47]. The second part in the superpotential comes from nonperturbative effects sourced by moduli stabilizing D7-branes wrapping certain four-cycles in the compactification and  $a = 2\pi/n$  is a constant with  $n$  denoting the number of wrapped branes. The inflaton dependence of the superpotential is induced by the interaction between the inflationary D3-brane and wrapped D7-brane. The displacement of the D3-brane in the compactification slightly modifies the supergravity background, perturbing the warp factor,  $h = h_{\text{KS}} + \delta h$ , which corrects the warped volume of four-cycles from D7-branes. This then corrects the D3-D7 potential inducing the backreaction on the mobile D3-brane. The prefactor in the nonperturbative part of (3.19) computed from corrections to the warped background takes the form [13]:

$$A(z^\alpha) = A_0 \left[ \frac{f(z^\alpha)}{f(0)} \right]^{1/n}, \quad (3.20)$$

where  $f(z^\alpha)$  denotes the holomorphic embedding function of D7-branes depending on the D3-brane coordinates and  $A_0$  is a constant.

According to this and (3.17) - (3.18), the scalar potential (3.11) takes the form [8, 14]

$$V_F(z^\alpha, \bar{z}^\alpha, \rho, \bar{\rho}) = \frac{\kappa^2}{3U(z, \rho)^2} \left\{ \left[ U(z, \rho) + \gamma k^{\gamma\bar{\delta}} k_\gamma k_{\bar{\delta}} \right] |W_{,\rho}|^2 - 3(\overline{W}W_{,\rho} + c.c.) \right\} \\ + \frac{\kappa^2}{3U(z, \rho)^2} \left[ \left( k^{\alpha\bar{\delta}} k_{\bar{\delta}} \overline{W}_{,\bar{\rho}} W_{,\alpha} + c.c. \right) + \frac{1}{\gamma} k^{\alpha\bar{\beta}} W_{,\alpha} \overline{W}_{,\bar{\beta}} \right]. \quad (3.21)$$

The first part in Eq. (3.21) is the standard KKLTT F-term potential, and the second part arises exclusively from corrections to the nonperturbative superpotential. By considering appropriate formulas for the various terms in the potential (e.g. see [12]), it is straightforward to show that the F-term potential (3.21) takes the functional form  $V_F = V_F(z_1 + \bar{z}_1, |z_1|^2, \eta, \sigma, \chi)$ . To obtain the explicit form of the potential, we need to specify an embedding for D7-branes. For the Kuperstein embedding of D7-branes [40], we have

$$f(z_1) = \mu - z_1 = 0. \quad (3.22)$$

Thus the prefactor of the nonperturbative superpotential, Eq. (3.20), and its derivative with respect to the independent coordinates take the form

$$A(z_1) = A_0 \left( 1 - \frac{z_1}{\mu} \right)^{1/n}, \quad (3.23)$$

$$A_i(z_1) = -\frac{A_0}{n\mu} \left(1 - \frac{z_1}{\mu}\right)^{1/n-1} \delta_{i1}. \quad (3.24)$$

To obtain a simple trajectory depending only on one harmonic mode, say  $\theta_1 = \theta$ , we fix the rest of angular directions of  $T^{1,1}$  by imposing the following constraints

$$\begin{aligned} \frac{\varphi_1 - \varphi_2 \pm \psi}{2} &= \pm \frac{\pi}{2}, & \frac{\varphi_1 + \varphi_2 + \psi}{2} &= \pi, \\ \frac{\psi - \varphi_1 - \varphi_2}{2} &= 0, & \theta_2 &= 0. \end{aligned} \quad (3.25)$$

Accordingly to this, the coordinates on the deformed conifold read as

$$\begin{aligned} z_1 &= -\epsilon \cosh\left(\frac{\eta}{2}\right) \cos\left(\frac{\theta}{2}\right), & z_2 &= -i\epsilon \sinh\left(\frac{\eta}{2}\right) \cos\left(\frac{\theta}{2}\right), \\ z_3 &= -i\epsilon \sinh\left(\frac{\eta}{2}\right) \sin\left(\frac{\theta}{2}\right), & z_4 &= +\epsilon \cosh\left(\frac{\eta}{2}\right) \sin\left(\frac{\theta}{2}\right). \end{aligned} \quad (3.26)$$

The imaginary part of the Kähler modulus can be integrated out by

$$\frac{e^{ia\chi}}{A} \rightarrow \frac{1}{|A|}. \quad (3.27)$$

Accordingly, the four-dimensional supergravity potential is<sup>1</sup>

$$\begin{aligned} V_F &= \frac{2\kappa^2 a_n^2 |A_0|^2 e^{-2a_n \sigma}}{U^2} |g(\eta, \theta)|^{2/n} \\ &\times \left\{ \frac{U}{6} + \frac{1}{a_n} \left( 1 - \frac{|W_0|}{|A_0|} \frac{e^{a_n \sigma}}{g(\eta, \theta)^{1/n}} \right) + F(\eta, \theta) \right\}, \end{aligned} \quad (3.28)$$

where

$$\begin{aligned} F(\eta, \theta) &= \frac{\gamma}{4} \epsilon^{4/3} K^4 \sinh^2 \eta - \frac{\epsilon K^3}{an\mu g} \cos\left(\frac{\theta}{2}\right) \cos\left(\frac{\eta}{2}\right) \sinh^2\left(\frac{\eta}{2}\right) \\ &+ \frac{\epsilon^{2/3} K^2}{4n^2 a^2 \mu^2 \gamma g^2} \left[ \sinh^2\left(\frac{\eta}{2}\right) \cos^2\left(\frac{\theta}{2}\right) + \frac{2}{3K^2} \sin^2\left(\frac{\theta}{2}\right) \right]. \end{aligned} \quad (3.29)$$

Here we have defined

$$U(\eta, \theta) = 2\sigma(\eta, \theta) - \gamma k(\eta), \quad (3.30)$$

---

<sup>1</sup>It is straightforward to check that for  $\theta = 0$  our two-field potential (3.28) coincides with the single field potential derived in [12].

$$g(\eta, \theta) = 1 + \frac{\epsilon}{\mu} \cosh\left(\frac{\eta}{2}\right) \cos\left(\frac{\theta}{2}\right). \quad (3.31)$$

Compactifying the warped deformed conifold throat via attaching it to the compact Calabi-Yau space at some finite radius  $r_{UV}$  requires Kähler modulus stabilization. This involves integrating out  $\sigma$  by the assumption that it evolve adiabatically while remaining in its instantaneous minimum  $\sigma_*(\eta, \theta)$  using the following equation:

$$\left. \frac{\partial(V_F + V_{\text{uplift}})(\eta, \theta)}{\partial\sigma} \right|_{\sigma_*(\eta, \theta)} = 0. \quad (3.32)$$

Here we have added a D-term potential  $V(\eta, \sigma) = D_{\text{uplift}}/U(\eta, \sigma)^2$  which is needed to uplift KKLT AdS minimum to a dS minimum. The uplifting can be sourced by distant anti-D3-branes or wrapped D7-branes. The instantaneous minimum of  $\sigma$  is denoted by  $\sigma_*$  including a shift due to its coordinate dependence induced by adding a mobile D3-brane to the compactification. The functional form of  $\sigma_*$  can be determined by the numerical solution of the transcendental equation (3.32), which we will solve in the next section. Before turning to numerical computation, we need to look at the minimum of the D3-D7 potential which specifies  $\sigma_F$  and  $D_{\text{uplift}}$  and constrains the rest of parameters on which these depend.

The critical value  $\sigma_F$  of the Kähler modulus before uplifting is determined by the condition  $D_\sigma W|_{\eta=0, \theta=0, \sigma_F} = 0$ , or equivalently [2],

$$e^{a\sigma_F} = \frac{|A_0|}{|W_0|} \left(1 + \frac{2}{3}a\sigma_F\right) g(0, 0)^{1/n} \quad \Rightarrow \quad \left. \frac{\partial V_F}{\partial\sigma} \right|_{\sigma_F} = 0, \quad (3.33)$$

where  $g$  is given by (3.31). We may write this in the form:

$$\sigma_F = \frac{1}{a} \log \left[ \frac{|A_0|}{|W_0|} \left(1 + \frac{2}{3}a\sigma_F\right) \left(1 + \frac{\epsilon}{\mu}\right)^{1/n} \right]. \quad (3.34)$$

From Eq. (3.28) and Eq. (3.33) we note that

$$V_F(0, \sigma_F) = -\frac{3a^2\kappa^2 W_0^2}{2\sigma_F(3 + 2a\sigma_F)^2}. \quad (3.35)$$

The uplifting parameter is given by

$$s = \frac{V_{\text{uplift}}(0, \sigma_F)}{|V_F(0, \sigma_F)|} \quad \text{with} \quad V_{\text{uplift}}(0, \sigma_F) = \frac{D_{\text{uplift}}}{4\sigma_F^2}. \quad (3.36)$$

From this we obtain

$$D_{\text{uplift}} = \frac{6s a^2 \kappa^2 W_0^2 \sigma_F}{(3 + 2a\sigma_F)^2}. \quad (3.37)$$

Here  $1 \leq s \leq 3$  to avoid decompactification,  $a$  is determined by the choice of  $n$ ,  $\kappa$  depends on the UV cut-off, and the value of  $\sigma_F$  can be derived from Eq. (3.34) once the set of parameters  $\{\epsilon, \mu, n, s, |A_0|, |W_0|\}$  are specified. We also note that uplifting the KKLT AdS minimum to a dS minimum introduces a small shift in the stabilized volume,  $\sigma_0 \equiv \sigma_F + \delta\sigma$ . At the tip, we have [15]:

$$a\sigma_0 \approx a\sigma_F + \frac{s}{a\sigma_F}. \quad (3.38)$$

Here we note that both Eq. (3.34) and Eq. (3.38) which give  $\sigma_F$  and  $\delta\sigma$ , respectively, are derived from the local minimum of the F-term potential (3.28) at the tip. The critical value  $\sigma_m$  of the Kähler modulus away from the tip and its shift  $\delta\sigma_m$  can be derived from the global minimum of the F-term potential (3.28). By computing the first and the second derivatives of the F-term potential  $V_F$  with respect to  $\sigma$ , it is straightforward to show that the global minimum of  $V_F$  requires:

$$\sigma_m = \frac{1}{a} \left[ g(\eta, \theta)^{1/n} \frac{|A_0|}{|W_0|} \left( 1 + \frac{aU}{3} + 2aF \frac{aU+2}{aU+4} \right) \right]. \quad (3.39)$$

Under the assumption that  $V_D$  only shifts the minimum by a small amount gives:

$$\begin{aligned} V'(\sigma_m + \delta\sigma) &= V'_D(\sigma_m + \delta\sigma) + V'_F(\sigma_m + \delta\sigma) \simeq V'_D(\sigma_m) + \delta\sigma V''_F(\sigma_m) = 0 \\ \Rightarrow \quad \delta\sigma &= -\frac{V'_D(\sigma_m)}{V''_F(\sigma_m)} = \frac{4V'_D(\sigma_m)}{U_m V_F(\sigma_m)} \frac{V(\sigma_m)}{V''(\sigma_m)}. \end{aligned} \quad (3.41)$$

We would like to remark here that in most of the brane inflation models in the literature (e.g. see [12, 15]) the perturbative expansion of the potential is analysed in either the UV or the IR region. In these models  $\epsilon$  and  $\mu$  are considered to have similar order magnitude. However, we note that a consistent expansion along the entire throat including both the UV and IR regions requires a large hierarchy between  $\epsilon$  and  $\mu$ . In particular, the final piece of our potential term  $F(\eta, \theta)$  given by Eq. (3.29) scales as  $\epsilon^{-4/3}(\epsilon/\mu)^2$ . Unless the hierarchy between  $\epsilon$  and  $\mu$  is large this term will dominate the potential and destabilize the vacuum expansion. Moreover, the key point here is that in our DBI brane inflation set up the full potential contains a large leading order mass term for the radial coordinate and in order to keep the adiabatic approximation (3.32) valid the hierarchy between  $\epsilon$  and  $\mu$  has to be large, so that the mass generated for the Kähler modulus is much larger than the Hubble rate (see Section 4).

As mentioned above, apart from the shift induced by uplifting,  $\delta\sigma$ , the addition of a mobile D3-brane in the compactification induces a further shift in the Kähler modulus that depends on the coordinates of the brane,  $\sigma_*(\eta, \theta)$ . Thus the nonperturbatively generated D3-D7 potential about a dS minimum takes the form:



$$\begin{aligned}
V_F + V_D = & \frac{2\kappa^2 a_n^2 |A_0|^2 e^{-2a_n \sigma_*(\eta, \theta)}}{U[\eta, \sigma_*(\eta, \theta)]^2} |g(\eta, \theta)|^{2/n} \\
& \times \left\{ \frac{U[\eta, \sigma_*(\eta, \theta)]}{6} + \frac{1}{a_n} \left( 1 - \frac{|W_0|}{|A_0|} \frac{e^{a_n \sigma_*(\eta, \theta)}}{g(\eta, \theta)^{1/n}} \right) + F(\eta, \theta) \right\} \\
& + \frac{D_{\text{uplift}}}{U[\eta, \sigma_*(\eta, \theta)]^2}.
\end{aligned} \tag{3.42}$$

Hence the total potential takes the form:

$$\begin{aligned}
T_3 V = & T_3 \left( \frac{1}{2} m_0^2 [r(\eta)^2 + c_2 K(\eta) \sinh \eta \cos \vartheta] + V_0 \right) \\
& + V_F + V_D.
\end{aligned} \tag{3.43}$$

Here  $D_{\text{uplift}}$  is given by (3.37), the constant  $V_0$  is chosen so that the global minimum of  $V$  is  $V = 0$ , and  $c_2$  is an arbitrary constant. Because  $c_2$  multiplies a solution to a free Laplace equation, it is not fixed. For a self-consistent expansion, we expect  $c_2$  to be smaller, or of a magnitude comparable with other terms in the potential.

Finally, to derive the explicit form of the D3-brane equations of motion on the deformed conifold for the simplest case including only one angular direction for the potential (3.43) first note that a simple  $S^3$  round metric on the deformed conifold can be obtained from (2.16) as:

$$ds^2 = A(\eta) d\eta^2 + B(\eta) d\theta^2, \tag{3.44}$$

where

$$A(\eta) = \frac{\epsilon^{4/3}}{6K(\eta)^2}, \quad B(\eta) = \frac{\epsilon^{4/3} K(\eta)}{4} [\cosh(\eta/2) + \sinh(\eta/2)]. \tag{3.45}$$

The brane equations of motion can then be derived upon cross elimination from (3.7) in the following form:

$$\begin{aligned}
\ddot{\eta} = & -\frac{3H}{\gamma_{\text{DBI}}^2} \dot{\eta} + \frac{h'}{\gamma_{\text{DBI}} h} \dot{\eta}^2 (1 - \gamma_{\text{DBI}}) + \frac{h'}{2h^2 A} (\gamma_{\text{DBI}}^{-1} - 1)^2 \\
& - \frac{1}{2A} (A' \dot{\eta}^2 - B' \dot{\theta}^2) + h \dot{\eta} \frac{V_\theta}{\gamma_{\text{DBI}} T_3} - (1 - hA \dot{\eta}^2) \frac{V_\eta}{\gamma_{\text{DBI}} A T_3} \\
& - \dot{\eta} \dot{\theta} (1 - \gamma_{\text{DBI}}^{-1}) \frac{h_\theta}{h},
\end{aligned} \tag{3.46}$$

$$\ddot{\theta} = -\frac{3H\dot{\theta}}{\gamma_{\text{DBI}}^2} + (1 - \gamma_{\text{DBI}})\dot{\theta}\dot{\eta}\frac{h'}{\gamma_{\text{DBI}}h} - \dot{\theta}\dot{\eta}\frac{B'}{B} + h\dot{\theta}\dot{\eta}\frac{V_\eta}{\gamma_{\text{DBI}}T_3} - (1 - hB\dot{\theta}^2)\frac{V_\theta}{\gamma_{\text{DBI}}BT_3} - (1 - \gamma_{\text{DBI}}^{-1})\left[\dot{\theta}^2 - \frac{(1 - \gamma_{\text{DBI}}^{-1})}{2hB}\right]\frac{h_\theta}{h}. \quad (3.47)$$

Here we note that in the absence of non-linear corrections including the contribution of the D3-D7 potential and perturbations of the warp factor, the potential (3.43) and Eqs. (3.46) - (3.47) reduce to the D3-brane potential and equations of motion including only linearized corrections studied in [32]. We also note that in the slow-roll regime  $\gamma_{\text{DBI}} \simeq 1$  and the perturbations of the warp factor have no effect. However, since we are neither slow-rolling nor restricting ourselves to the linearized case, we need to take into account the perturbations of the warp factor which amount super-potential corrections in the D3-D7 potential. For the warped deformed conifold, the perturbations of the warp factor in the tip region can be expressed as [39]

$$\begin{aligned} \delta h &= -(2\pi)^4 g_s p(\alpha')^2 \mathcal{G}(\eta, y_4) \\ &= \frac{16\pi \cdot 3^{2/3} g_s p(\alpha')^2}{(2\epsilon)^{8/3} \cdot \eta} \left[ 1 + 2\sqrt{2}y_4 + 6y_4^2 + 8\sqrt{2}y_4^3 - \dots \right] \quad \text{with} \end{aligned} \quad (3.48)$$

$$y_4 = \left[ \cos\left(\frac{\theta_1}{2}\right) \sin\left(\frac{\theta_2}{2}\right) e^{\frac{i(\varphi_1 - \varphi_2 + \psi)}{2}} - \sin\left(\frac{\theta_1}{2}\right) \cos\left(\frac{\theta_2}{2}\right) e^{\frac{-i(\varphi_1 - \varphi_2 - \psi)}{2}} \right]. \quad (3.49)$$

Here  $\mathcal{G}(\eta, y_4)$  is the Green's function (expanded in the eigenfunctions of the Laplacian) on the deformed conifold given by (2.31),  $y_4$  comes from (2.12) and  $p$  specifies the number of mobile D3-branes, which we take to be  $p = 1$ . By the first angular condition in (3.25) along an  $S^3$  ( $\theta_1 = 0$ ) for relations (3.48) and (3.49) we have

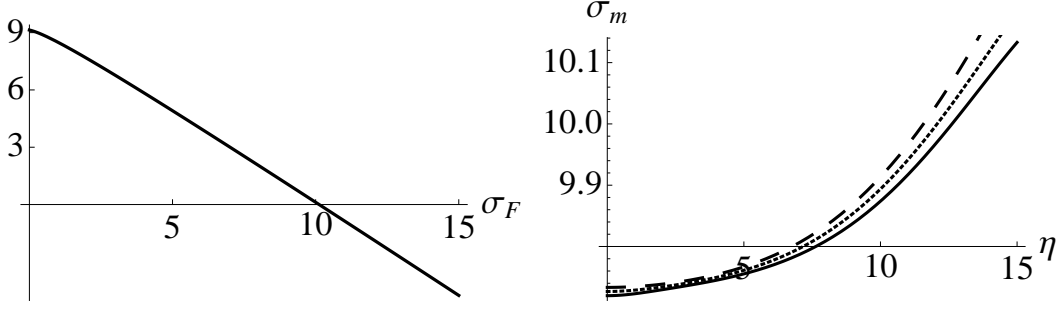
$$\delta h = \frac{16\pi \cdot 3^{2/3} g_s p(\alpha')^2}{(2\epsilon)^{8/3} \cdot \eta} \left[ 1 + 2\sqrt{2}y_4 + 6y_4^2 + 8\sqrt{2}y_4^3 - \dots \right] \quad \text{with} \quad (3.50)$$

$$y_4 = -\frac{1}{\sqrt{2}} \sin\left(\frac{\theta}{2}\right). \quad (3.51)$$

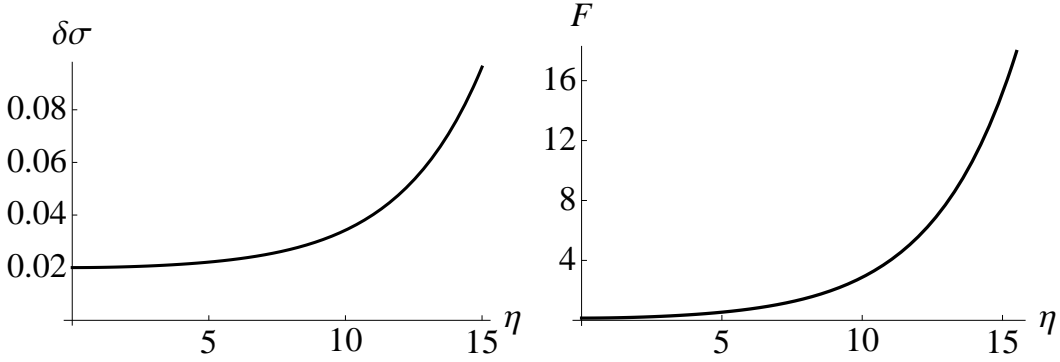
In the off-tip region, where  $r$  is large, we may use the very first line in (3.48) together with relations (2.33) - (2.37) for the quantum numbers  $j_1 = j_2 = R/2 = 1/2$  and  $m_1 = m_2 = 1/2$ , and obtain  $\delta h$  in the form

$$\delta h \simeq -\frac{10g_s p \alpha'^2}{\pi^2} \cos^4 \frac{\theta}{2} \left( \frac{1}{r^4} \right) + \dots \quad \text{with} \quad (3.52)$$

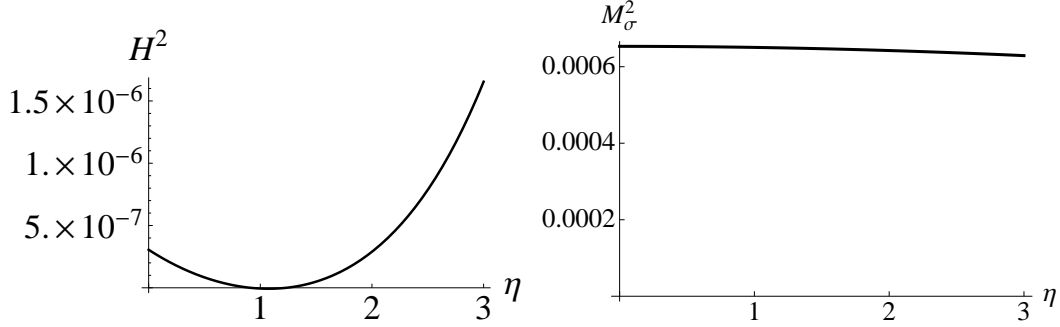
$$r^3 = \epsilon^2 e^\eta. \quad (3.53)$$



**Figure 1:** The behaviour of  $\sigma_F$  for choice of compactification parameters (4.1), and the behaviour of  $\sigma_m$  for choice of parameters (4.1) with  $\theta = 0$  (solid),  $\pi/2$  (tiny-dashed) and  $\pi$  (large-dashed).



**Figure 2:** The behaviour of the function  $F$  and  $\delta\sigma$  for the choice of parameters (4.1) and  $\theta = \pi$ .



**Figure 3:** The Hubble rate,  $H^2 \simeq V/3M_{\text{pl}}^2$ , and the  $\sigma$  mass squared,  $M_\sigma^2$  for the choice of parameters (4.1) and  $\theta = \pi$ .

## 4. Inflationary solutions

In order to integrate the full brane equations of motion, we first need to specify a suitable choice of parameters and then compute the real part of the Kähler modulus that appears in the brane equations of motion. To choose a reasonable set of compactification parameters, we note the following points. Firstly, we require a large hierarchy between  $A_0$  and  $W_0$  to guarantee large  $\sigma_F$  which ensures suppressed  $\alpha'$ -corrections.

Secondly, we also need a large hierarchy between  $\epsilon$  and  $\mu$  in addition to the  $A_0/W_0$  hierarchy to guarantee a valid perturbative expansion,  $\delta\sigma \ll 0$ . When both these hierarchies are turned on,  $\sigma_*$  can be computed within the adiabatic approximation from Eq. (3.32). We also remark from the literature that choosing a large value of the UV-scale,  $\eta_{UV}$ , sets a large value for the Planck-mass (e.g. see [32]) in which case curvature corrections may be omitted and non-linear corrections are dominated by IASD fluxes sourced by moduli stabilizing wrapped D7-branes whose number is given by  $n > 1$ . Furthermore, we remark that the supergravity solution requires large  $g_s M$  and the value of  $s$  has to be chosen within the range  $1 \leq s \leq 3$  to ensure a small positive cosmological constant and to avoid runaway decompactification.

In line with the above requirements we choose in our numerical analysis the following specific set of compactification parameters:

$$\begin{aligned} \eta_{UV} = 15, \quad n = 2, \quad s = 2, \quad \epsilon = 0.001 \\ g_s M = 100, \quad \mu = 5, \quad W_0 = 29, \quad A_0 = 25 \times 10^{12}. \end{aligned} \quad (4.1)$$

Inspection of Eq. (3.34) and Eq. (3.39) shows that for the choice of parameters (4.1)  $\sigma_F$  and  $\sigma_m$  scale to large values (see Fig. 1); inspection of Eq. (3.29) and Eq. (3.41) shows that for the choice of parameters (4.1) and  $\theta = \pi$  the functions  $\delta\sigma$  and  $F$  scale to reasonably small values on the entire warped deformed conifold (see Fig. 2); for the choice of parameters (4.1) the mass squared of the Kähler modulus,  $M_\sigma^2 \equiv V_{,\sigma\sigma}$ , is much larger than the approximate squared Hubble rate,  $H^2 \simeq V/3M_{\text{pl}}^2$  (see Fig. 3), which guarantees an adiabatic expansion [15, 48]. We note here that by decreasing these hierarchies much below their values (4.1) makes  $M_\sigma^2$  and  $H^2$  scale similarly which invalidates the adiabatic approximation, in addition  $\delta\sigma$  and  $F$  become large which invalidates the perturbative expansion.

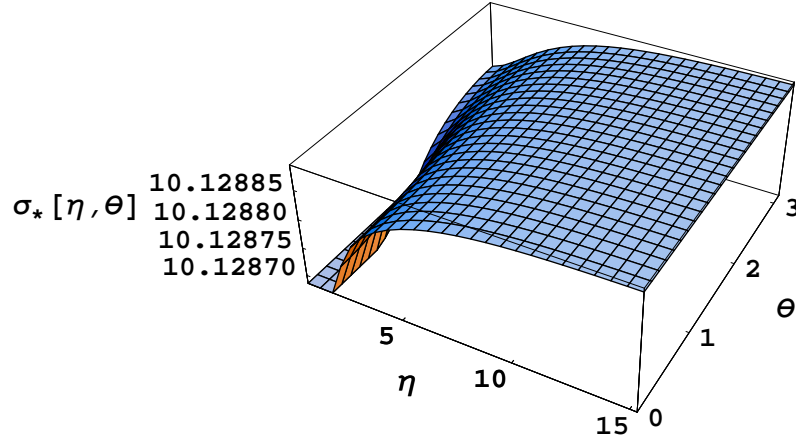
In the literature of brane inflation, the most common way of computing the Kähler modulus,  $\sigma$ , is to adopt a semi-analytic approach with the assumption that  $\sigma$  evolves adiabatically (e.g. see [14]). In this approach,  $\sigma$  in  $U(\eta, \sigma)$  is set to its large fixed value  $\sigma_0$ , and (3.32) is treated as an equation in the variable  $\exp[-a\sigma_*(\eta)]$ . The value of  $\sigma_*$  is then obtained from Eq. (3.32) by expanding in specific conical regions including either the IR or the UV regions where the canonical inflaton (3.10) is given by its small or large  $\eta$  limit, respectively. This may be a good approximation but it gives only a qualitative understanding. Here we compute  $\sigma_*$  by solving the transcendental equation (3.32) numerically to obtain the exact value of  $\sigma_*$  on the entire supergravity background. For the choice of parameters (4.1) we show  $\sigma_*$  in Fig. 4.

For the choice of parameters (4.1) and the numerically computed Kähler modulus, we integrated the full D3-brane equations of motion, Eqs. (3.5)-(3.7) with Eq. (3.7) given by Eqs. (3.46)-(3.47), and our inflationary solution is displayed in

Fig. 5. The solution describes spiral brane motion at high speed in the warped throat region of the compact Calabi-Yau space containing holomorphically embedded wrapped D7-branes involved in (Kähler) moduli stabilization. The conserved angular momentum is lifted by harmonic dependent corrections from linearized as well as non-linear perturbations including contributions from the D3-D7 potential and corrections of the warp factor. The brane accelerates along the radial and angular directions as it falls down the throat from the UV end where it is attached to the compact Calabi-Yau space. Inflation ends when the brane reaches the IR location where the throat smoothly closes off. For the choice of parameters and the numerically computed Kähler modulus, we integrated the brane equations of motion and found that the inflationary solution is quite robust against harmonic dependent corrections from the D3-D7 potential and corrections of the warp factor. In particular, we found (as displayed in Fig. 5) that harmonic dependent corrections induced by the D3-D7 potential and perturbations of the warp factor do not have the effect of increasing the number of e-foldings and decreasing the  $\gamma_{\text{DBI}}$ -factor. This result differs from our previous results [32] in which harmonic dependent correction to brane motion from linearized perturbations of the supergravity solution increased the number of e-foldings compared to the number of e-foldings produced by brane motion with conserved angular momentum (spinflation) where no supergravity corrections and hence no harmonic dependence in brane motion is present.

We repeated the above computation for a various choices of initial conditions and compactification parameters and our findings are summarized as follows.

- Decreasing  $\epsilon$  below its considered value while keeping other parameters fixed tends to flatten the functional form of  $\sigma_*$  and changes its overall scale insignificantly. Also, the decrease in  $\epsilon$  leaves  $\sigma_F$  unchanged.
- Decreasing the value of  $\mu$  while keeping other parameters fixed slightly decreases the value of  $\sigma_F$  and changes the scale of  $\sigma_*$  by a minimal amount. Note that here we decrease  $\mu$  by an amount, so that the hierarchy between  $\epsilon$  and  $\mu$  still remains large.
- Increasing/decreasing  $n$  while keeping other parameters fixed strongly impacts the  $\sigma_F$  and the scale of  $\sigma_*$ . Also, increasing  $n$  induces fluctuations in  $\sigma_*$  at large  $\eta$ .
- Increasing/decreasing the hierarchy between  $A_0$  and  $W_0$  while keeping other parameters fixed increases/decreases the value of  $\sigma_F$  and slightly changes the scale of  $\sigma_*$  but leaves its overall shape unchanged. Here again we do not decrease the hierarchy between  $A_0$  and  $W_0$  too much. Also, increasing the hierarchy between  $A_0$  and  $W_0$  by taking  $W_0$  much smaller than its considered value increases  $\sigma_F$  and suppresses the uplifting contribution. This slightly changes the form of  $\sigma_*$  near the origin.
- Changing the set of compactification parameters and initial conditions indicates that the number of e-foldings produced by spinflation including non-linear harmonic dependent corrections depends more on the subset of compactification parameters  $\{\epsilon, g_s M, \eta_{\text{UV}}\}$  than on the initial conditions and is insensitive to the choice of the remaning subset of compactification parameters  $\{n, s, \mu, A_0, W_0\}$ . Comparing with



**Figure 4:** The Kähler modulus,  $\sigma_*(\eta, \theta)$ , on the entire supergravity background with the choice of compactification parameters (4.1).

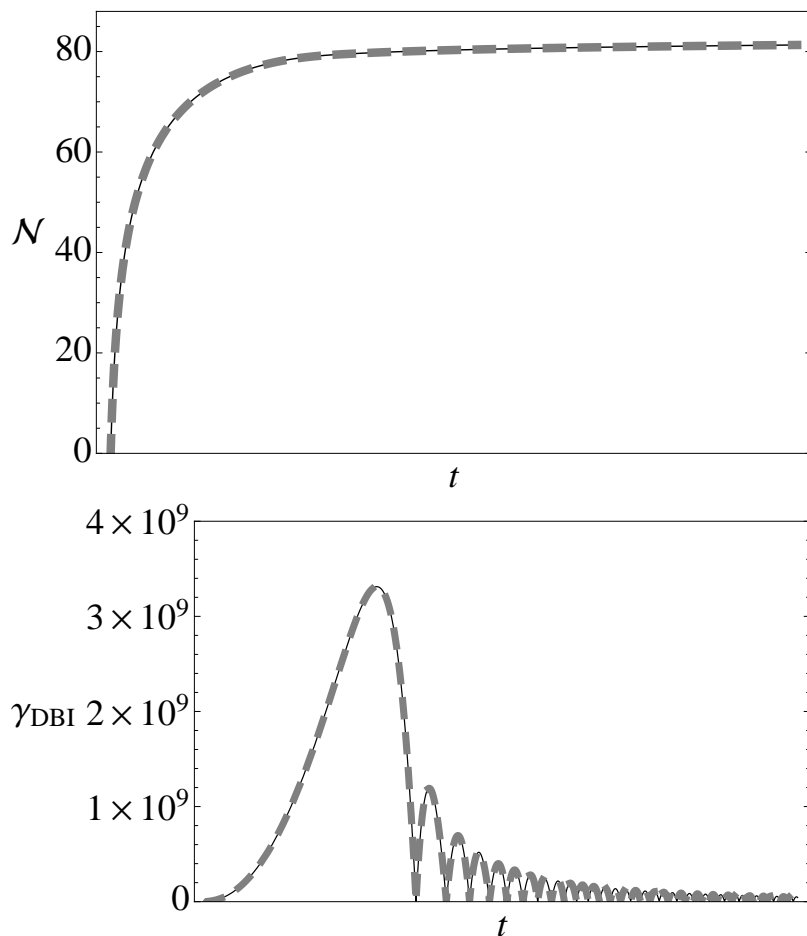
the number of e-foldings generated by spinflation including only linearized harmonic dependent corrections each time when varying the set of parameters and initial conditions shows no difference in the number of e-foldings (as Fig. 5).

- The  $\gamma_{\text{DBI}}$ -factor produced by spinflation including non-linear harmonic dependent corrections is insensitive to the choice of compactification parameters and initial conditions. Comparing with the  $\gamma_{\text{DBI}}$ -factor produced by spinflation with only linearized harmonic dependent corrections each time when varying the set of parameters and initial conditions shows no difference in the  $\gamma_{\text{DBI}}$ -factors (as Fig. 5).

The above findings show that the inflationary solution is quite robust against harmonic dependent corrections from the D3-D7 potential and perturbations of the warp factor not just for the specific choice of compactification parameters (4.1) but for a very large set of consistent parameters.

## 5. Summary and conclusions

In this paper we studied brane inflation in a warped string compactification incorporating the effects of moduli stabilization and backreaction from UV-deformations of the warped throat geometry. The focus of our paper was on DBI brane inflation in the warped deformed conifold with a UV/IR consistent perturbative expansion around the noncompact ISD solution. The perturbations were dominated by IASD fluxes sourced by moduli stabilizing wrapped D7-branes.



**Figure 5:** The number of e-foldings,  $\mathcal{N}$ , and the  $\gamma$ -factor,  $\gamma_{\text{DBI}}$ , with (gray-dashed) and without (black-solid) non-linear harmonic dependent corrections for the choice of compactification parameters (4.1).

We computed the D3-brane potential on the entire deformed conifold including non-linear corrections from the flux induced potential in ten-dimensional supergravity which equals the nonperturbatively generated D3-D7 brane potential in four-dimensional supergravity. For a simple choice of a trajectory on the deformed conifold, we integrated out the Kähler modulus and reduced the D3-brane potential to a simple two-field potential depending on radial and harmonic directions of the deformed conifold. We integrated out the Kähler modulus by full numerical computation determining its exact functional form on the entire supergravity background including both the IR and UV regions. We found that a UV/IR consistent perturbative expansion in the supergravity potential with the Kähler modulus integrated out within the adiabatic approach in DBI inflation requires certain hierarchies of scales that determine the set of compactification parameters different from those in slow-roll models.

For the consistent choice of parameters and the numerically computed Kähler modulus, we integrated the D3-brane equations of motion in the warped deformed conifold with harmonic dependence from the D3-brane potential and perturbations of the warp factor. We found that our numerical solutions are quite robust against non-linear perturbations including harmonic dependent corrections from perturbations of the warp factor and the D3-D7 brane potential. In particular, we found that harmonic dependent corrections from the D3-D7 potential and perturbations of the warp factor do not have the effect of increasing the number of e-foldings and decreasing the  $\gamma_{\text{DBI}}$ -factor. We therefore conclude that the most leading order harmonic dependent correction to brane (spin)inflation comes from the linearized corrections analysed in [32] with the level of non-Gaussianity remaining large.

Our analysis can be extended in several ways. One direction would be to consider different embedding functions for D7-branes with different trajectories on the deformed conifold and see how this may affect the inflationary solutions. Despite the fact that different embedding functions for D7-branes and different trajectories on the deformed conifold modify the functional form of the supergravity potential, the Kähler modulus and perturbations of the warp factor, we expect this to have a subdominant effect on the number of e-foldings and the  $\gamma_{\text{DBI}}$ -factor. It would also be interesting to consider the possibility of a dynamical Kähler modulus (instead of stabilized) [48], and integrate the brane equations of motion for a less restricted choice of parameters. Since taking the Kähler modulus field dynamical adds to the number of brane equations of motion, we expect this to have a less trivial impact on the inflationary solutions, though we do not expect this to decrease the  $\gamma_{\text{DBI}}$ -factor by an appreciable amount.

The other, perhaps more interesting way of extending our analysis is to consider further corrections to the inflaton action and analyse in detail the effects of cosmological perturbation theory. One particular correction comes from the contribution of the flux induced potential of harmonic type [18]. The flux induced D3-D7 potential that we considered in our inflationary analysis came from the holomorphic solution of the noncompact supergravity equation of motion which described non-linear perturbations around the ISD solution. Since the general solution should be harmonic rather than just holomorphic it would be necessary to include such harmonic contributions which have not been computed for the deformed conifold to date.

Another correction to the inflaton action arises from departures of the noncompact limit [18] considered in this paper. Despite the fact that our Planck-mass was set large by the UV-scale we considered, it would be interesting to consider possible departures from the noncompact limit and compute further contributions to the D3-brane potential from coupling to curvature corrections which also induce harmonic dependence in brane motion [18]. In particular, coupling to the Ricci-scalar introduces a non-minimal coupling in the DBI action which corrects the  $\gamma_{\text{DBI}}$ -factor and may have the capacity of decreasing the level of non-Gaussianity significantly [49].



It would be interesting to confirm this result in the concrete supergravity set up considered in this paper and investigate its implications for cosmological perturbations in more detail [50] in our framework.

Finally, it would be important to confirm whether in our supergravity set up multifield effects (e.g. from phase transition) induced by an instability along harmonic directions [51] do have the capacity to evade stringent constraints in cosmological perturbations for single field inflationary models. Moreover, it would be very interesting, though formidable, to consider our DBI brane inflation model along a trajectory on the deformed conifold depending on all six directions and extract the full multifield effects for cosmological perturbations in a UV/IR consistent expansion and make contact with some of the results obtained in [52] by taking the singular conifold limit. We shall leave the investigation of these for the future.

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